Metrology in Electricity

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Metrology in Electricity

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1. Introduction

What is metrology? The word is used world-wide, but the meaning differs in the various languages. The International Organisation for Standardisation (ISO) gives the following definition:

Metrology is the field of knowledge concerned with measurement.

Now, what is measurement? Once again ISO answers:

Measurement is the set of operations having the object of determining the value of a quantity.

A third question has to be answered: What is the value of a quantity? The answer is given by the definition:

The value of a quantity is the expression of a quantity in terms of a number and an appropriate unit of measurement.

We see from the three questions and their answers that the task of metrology is
- to develop appropriate units of measurement,
- to develop sets of operations for determining the values of quantities,
- to develop the field of knowledge concerned with measurement.

During the history of mankind people have used many different units. One example of the unit of length is shown in Fig. 1. The mean length of the feet of 16 male, adult persons, tall and short, returning from Sunday church-going, had been taken in medieval Switzerland as a standard of length, the "foot". Clearly this measurement standard has the advantage of not being bound to a certain location, but its reproducibility is poor and its relative uncertainty may be in the range of 10%, despite of applying the principle of averaging. Modern standards for the meter, the unit of length, provide a relatively small uncertainty in the range of 10^{-10}.

It is obvious that units should not be chosen arbitrarily but should form a system of units. International cooperation under the auspices of the Metre Convention of 1875 has led to a system based on the seven base units shown in Fig. 2, the International System of Units (SI). The derived units were chosen in a way that they can be expressed as a product of base units with integer exponents and where no factor other than 1 occurs. The fact that such a so-called "coherent system" was chosen was of great importance for the acceptance and propagation of the SI system all over the world.

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Fig. 1:
Medieval unit of length as the average of the feet of 16 men.

The definition of the SI base units changed several times, but the changes were such that the realisations following the more recent definitions were always within the uncertainty interval of the previous ones. It is worth mentioning that only the kilogram, the second and the kelvin are independent of the other base units.

A necessary prerequisite for world-wide benefit from an international system of units is not only its acceptance. Each measurement standard in use must also be traceable to the defined unit, so that the uncertainty of the measurement result becomes evident. This can be achieved through an unbroken chain of calibrations with national and international measurement standards.

2. Electric Units within the SI

As shown above, only one electric base unit, the ampere, was chosen, thus the electric and magnetic units depend on the mechanical units (Fig. 3). As can be seen from the indicated figures of relative uncertainty of realisation, the dissemination of electric units is very much less accurate than the dissemination of mechanical ones.

unit of	symbol	defined by	year
length	m	"The meter is the length of the path travelled by light in vacuum in the time interval 1/299 792 458 of a second"	1983
mass	kg	international prototype	1889
time	s	transition of ^{133} Cs-atom	1967
electric current	A	force between conductors	1948
thermodyn. temperature	K	triple point of water	1967
amount of substance	mol	particles in 0,012 kg of ^{12}C	1971
luminous intensity	cd	radiation of 1/683 W/sr	1979

Fig. 2:
SI base units.

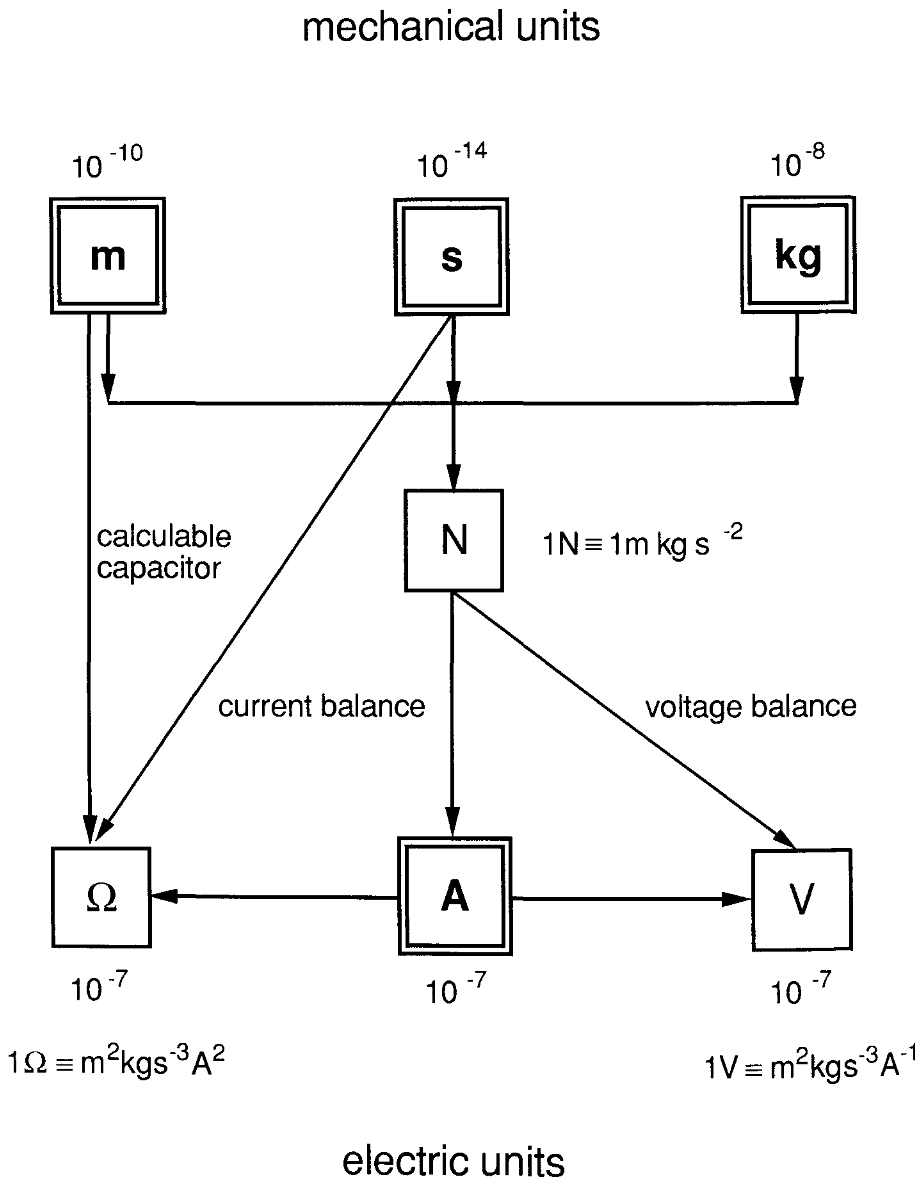


Fig. 3:
Relation between electric and mechanical units and relative uncertainty of their dissemination.

The sufficiently accurate realisation of the electric units according to their SI definition is time-consuming and requires costly experiments. Only big national laboratories can afford to spend so much time and money.

However, it is vital that at least two laboratories perform such fundamental experiments, because we need independent comparisons from time to time in order to further reduce the still large uncertainty of realisation of electric units. In the following some of the relevant developments are described.

2.1 The Realisation of the Volt by a Balance

Fig. 4 shows the principle of the voltage balance that has been realised at the Physikalisch-Technische Bundesanstalt (PTB), Germany [1]. An electrostatic force F_e between the electrodes of a cylindrical capacitor with capacitance C is compared with the gravitational force F_m acting on a mass m.

If one of the electrodes is moved vertically by a small value Δs, the mechanical work ΔW_m due to the displacement Δs is transformed into electric energy of the capacitor ΔW_e:

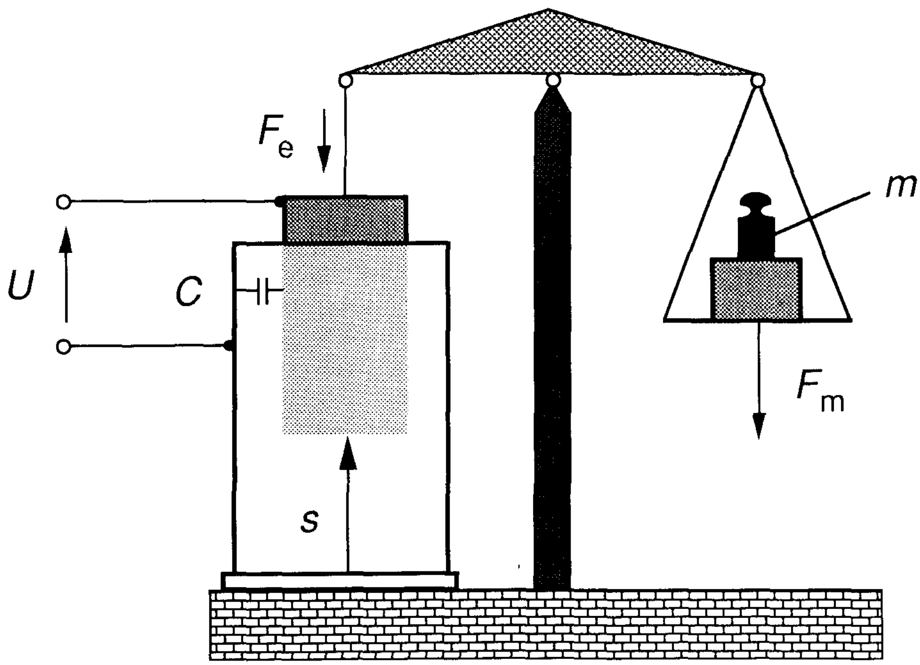


Fig. 4:
Principle of the voltage balance at the PTB.

$$\Delta W_m = F_m \, \Delta s = m \, g \, \Delta s$$

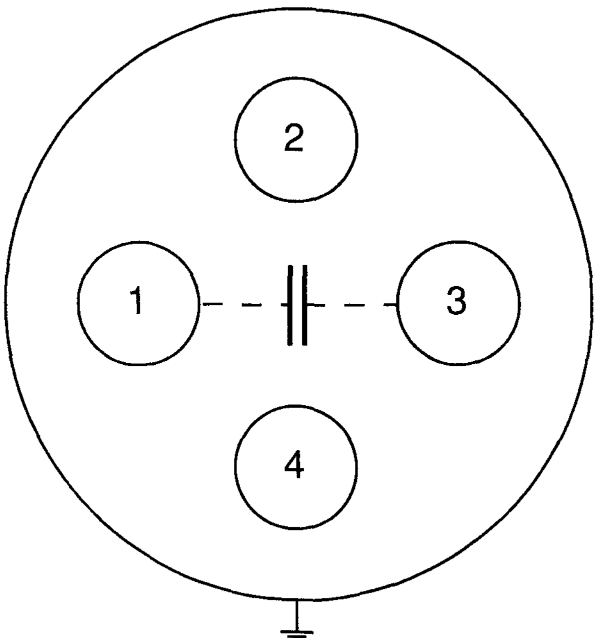
$$\Delta W_e = F_e \, \Delta s = -(1/2) \, (\mathrm{d}C/\mathrm{d}s) \, U^2 \, \Delta s$$

The capacitor is constructed in such a way that during the displacement the electric force stays constant, therefore $\mathrm{d}C/\mathrm{d}s = \Delta C/\Delta s$.

Assuming disconnection of the voltage source one obtains from the law of conservation of energy $\Delta W_m + \Delta W_e = 0$:

$$U = \sqrt{2 mg \, \Delta s/\Delta C.}$$

In order to determine the voltage of the capacitor, the mass m, the acceleration of gravity g at the location of the balance, the displacement Δs and the change in capacitance ΔC are measured. For our balance we chose $m = 2$ g, therefore $F_m = 20$ mN. The voltage is 10 000 V and is divided down to 1 V in order to be used as a standard. The change in capacitance can be calibrated via the calculable cross capacitor described below. The relative uncertainty of the PTB voltage balance is $3 \cdot 10^{-7}$.



$$\frac{C}{l} = \frac{\varepsilon_0}{\pi} \ln 2 = 1{,}953\,549\,043 \, \frac{\mathrm{pF}}{\mathrm{m}}$$

Fig. 5:
Principle of a cross-capacitor according to Thompson and Lampard.
1, 2, 3 and 4 are cylindrical electrodes.

2.2 The Realisation of the Ohm by a Calculable Cross Capacitor

The principle of a so-called cross capacitor is shown in Fig. 5. It consists of four cylindrical electrodes, and the capacitance between two opposite electrodes is measured using the neighbouring electrodes as guard electrodes.

In 1957, A.M. Thompson and D.G. Lampard showed that for any such cross capacitor the following relation is valid [2]:

$$\exp(-\pi \cdot C_{13}/(\varepsilon_0 \cdot l)) + \exp(-\pi \cdot C_{24}/(\varepsilon_0 \cdot l)) = 1$$

where C_{13} and C_{24} are the capacitances between electrodes 1 and 3 and 2 and 4 respectively and where l is the length of the electrodes. In the case of symmetry

$$C_{13} = C_{24} = C$$

we have

$$C/l = \varepsilon_0 \cdot \ln 2/\pi$$

This means that only one accurate length measurement is needed for the determination of C in SI units. C/l is roughly 2 pF/m.

On the basis of this theorem a symmetrical cross capacitor with vertical electrodes has been developed at PTB (Fig. 6). Through various comparisons of capacitances and impedances, and impedances and resistances, the farad and the ohm can be derived. The work is not yet finished, and the relative uncertainty is at present about 10^{-7}. At the National Institute of Standards and Technology (NIST), USA, this realisation of the ohm has been completed with a relative uncertainty of $2{,}2 \cdot 10^{-8}$ [3].

2.3 The Realisation of the Watt and the Ampere by the Moving Coil Balance

At the National Physical Laboratory (NPL), Great Britain, B. Kibble has developed an experiment [4] which allows the watt and – if a resistance is known through the calculable capacitor – the volt or the ampere to be determined (Fig. 7).

In the first step of the experiment, in which the set-up is used as a balance, the electric force on part of a specially constructed coil of length l in a magnetic field B carrying a current I is measured via a gravitational force. If the vector of B is orthogonal to the direction of I, we receive:

$$I \cdot \int B \mathrm{d}l = m \cdot g$$

In the second step of the experiment $\int B\mathrm{d}l$ is determined using the effect of electromagnetic induction. The coil is moved through the field B and the induced voltage U and the velocity v are measured

$$U = v \cdot \int B \mathrm{d}l$$

We then have:

$$U \cdot I = m \cdot g \cdot v$$

If a resistance R is known in SI units, the current I is fed through this resistance and $U = RI$ can be measured.

$$I = \sqrt{m \cdot g \cdot v/R}$$

and

$$U = \sqrt{m \cdot g \cdot v/R}$$

are then obtained.

This means that the moving coil experiment can also be used to realise the ampere and the volt. Neither the dimensions of the coil nor the flux density and distribution of the magnetic flux need to be measured, which illustrates the elegance of this method. Its present relative uncertainty is $3{,}5 \cdot 10^{-8}$.

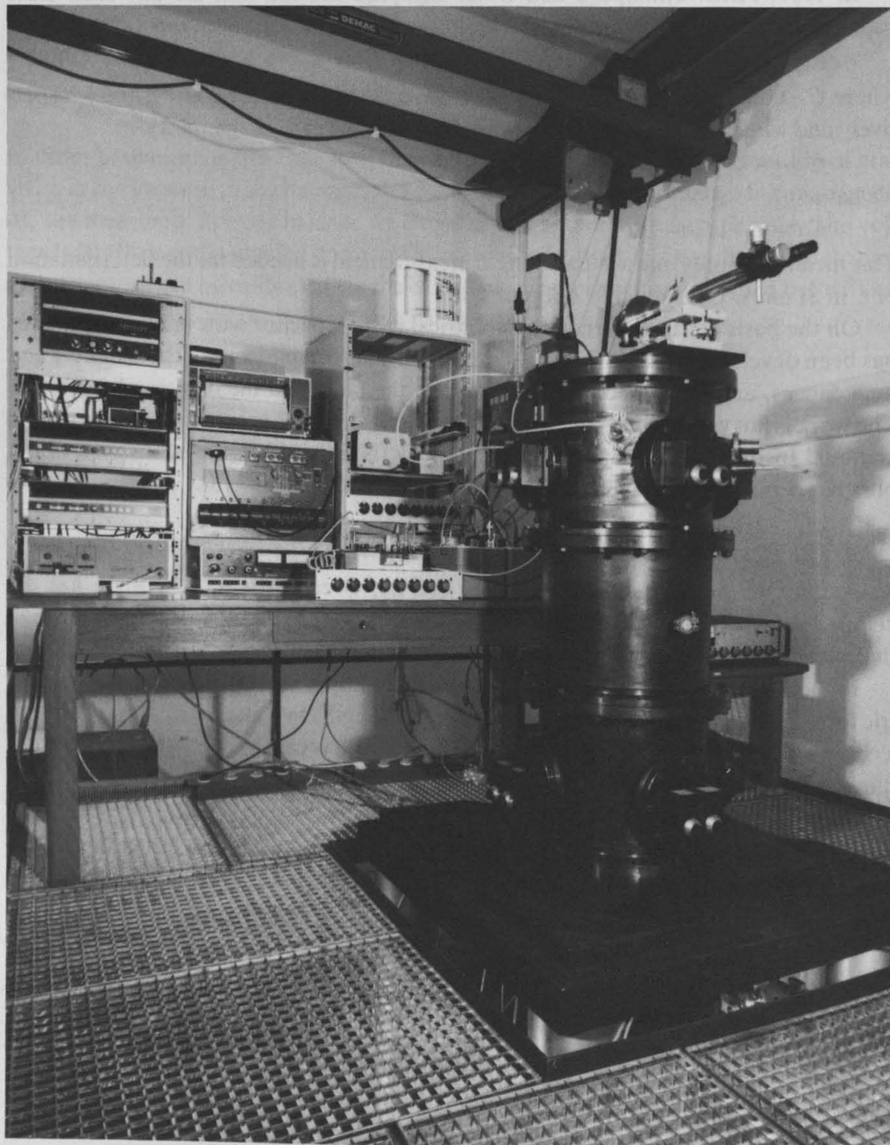


Fig. 6:
Cross capacitor at the PTB.

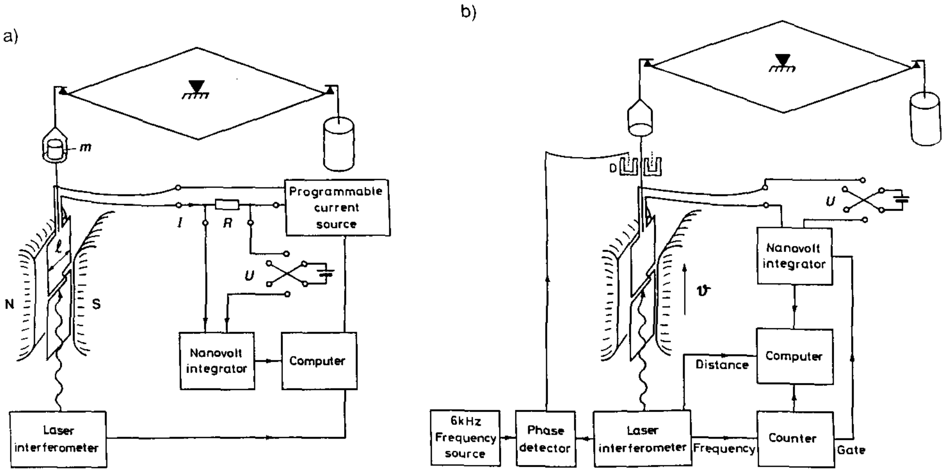


Fig. 7:
Schematic diagram of the moving-coil experiment for the determination of the ampere: a) force measurement, b) U/v measurement.

3. Macroscopic Quantum Effects

In 1963, a quantisation of voltage was observed for the first time, and in 1980 a quantisation of resistance. Effects like these are called macroscopic quantum effects. They describe effects where physical quantities which are observed on samples of macroscopic scale appear quantised – apart from their normal behaviour – and which can only be explained by the use of quantum mechanics.

The experimental conditions under which these effects may be observed are

– very low temperatures,

sometimes in addition

– high magnetic fields

and

– a specially prepared sample.

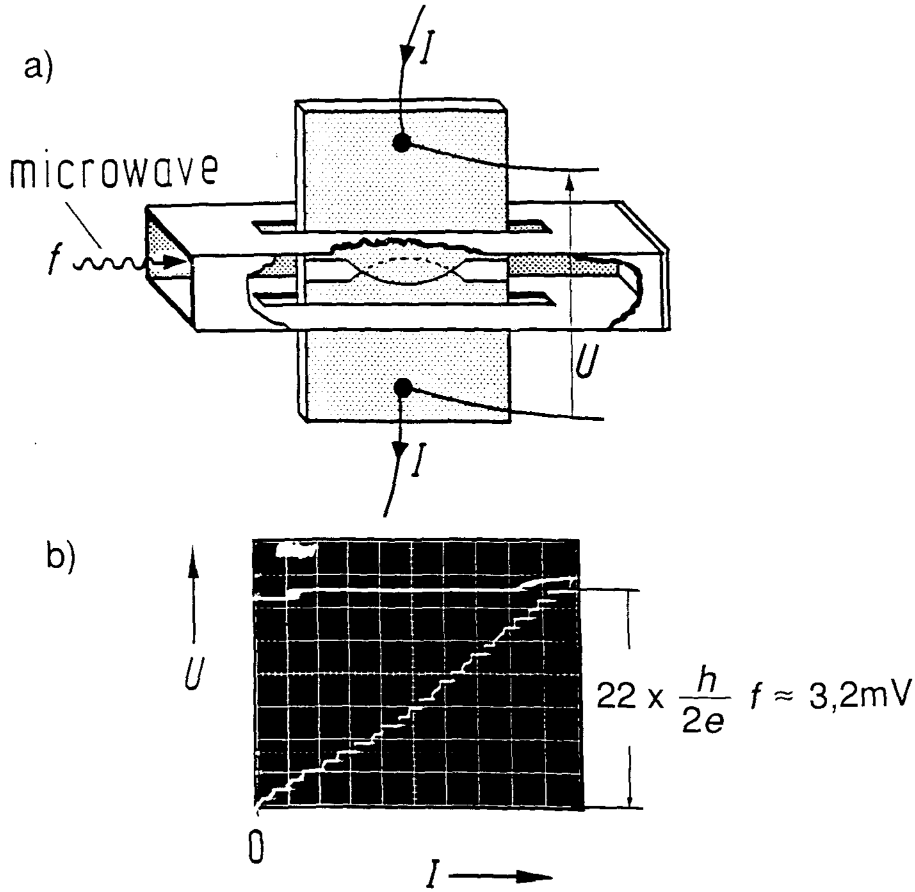
3.1 The AC Josephson Effect

This effect was predicted in 1962 by Brian D. Josephson [5] who was awarded the Nobel Prize in 1973 for his work. A device of two weakly-coupled superconductors is exposed to a direct current and an ac microwave field (Fig. 8). Under these conditions steps of constant voltage difference occur in the characteristic at

$$U(\mathrm{n}) = \mathrm{n} \cdot (h/2e) \cdot f, \ \mathrm{n} = 1, 2, \ldots$$

where f is the frequency of the ac microwave field and $h/2e$ is a characteristic fundamental constant known in SI units with a relative uncertainty of $4 \cdot 10^{-7}$.

Today it is possible to make a series connection of up to 20 000 of these Josephson junctions [6]. The total voltage that can be obtained from such a device ranges from −15 V to +15 V (Fig. 9).



$$\boxed{U(n) = n\,\frac{h}{2e}\,f}$$

Fig. 8:
Reproduction of the volt via the Josephson effect:
a) general arrangement of one junction in a wave guide
b) DC current-voltage characteristic for 70 GHz.

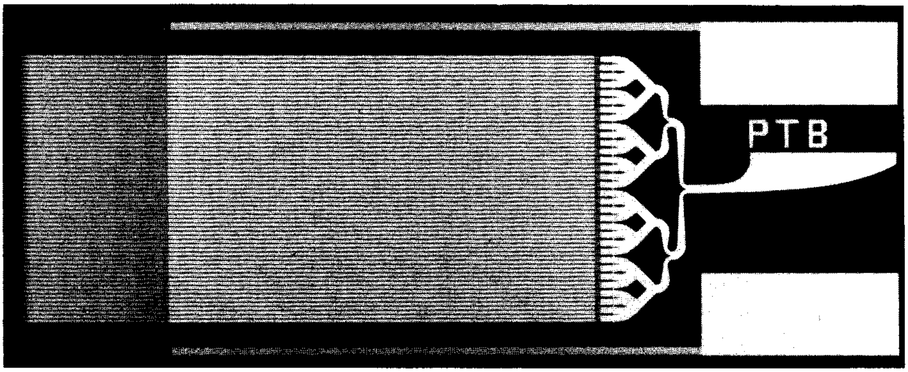


Fig. 9
10 V Josephson array standard chip with 20 160 junctions (chip size: 10 mm x 20 mm).

3.2 The Quantum Hall Effect

The quantum Hall effect (QHE) was discovered in 1980 by K. von Klitzing [7] who was awarded the Nobel Prize in 1985. Fig. 10 shows the principal set-up of a QHE sample and the characteristic plateaus of constant Hall resistance

$$R_H(i) = (1/i) \cdot (h/e^2),\ n = 1, 2, \ldots$$

where h/e^2 is again a characteristic fundamental constant which is known in SI units with a relative uncertainty of $2 \cdot 10^{-7}$. Before the discovery of the QHE no fixed points of resistances existed.

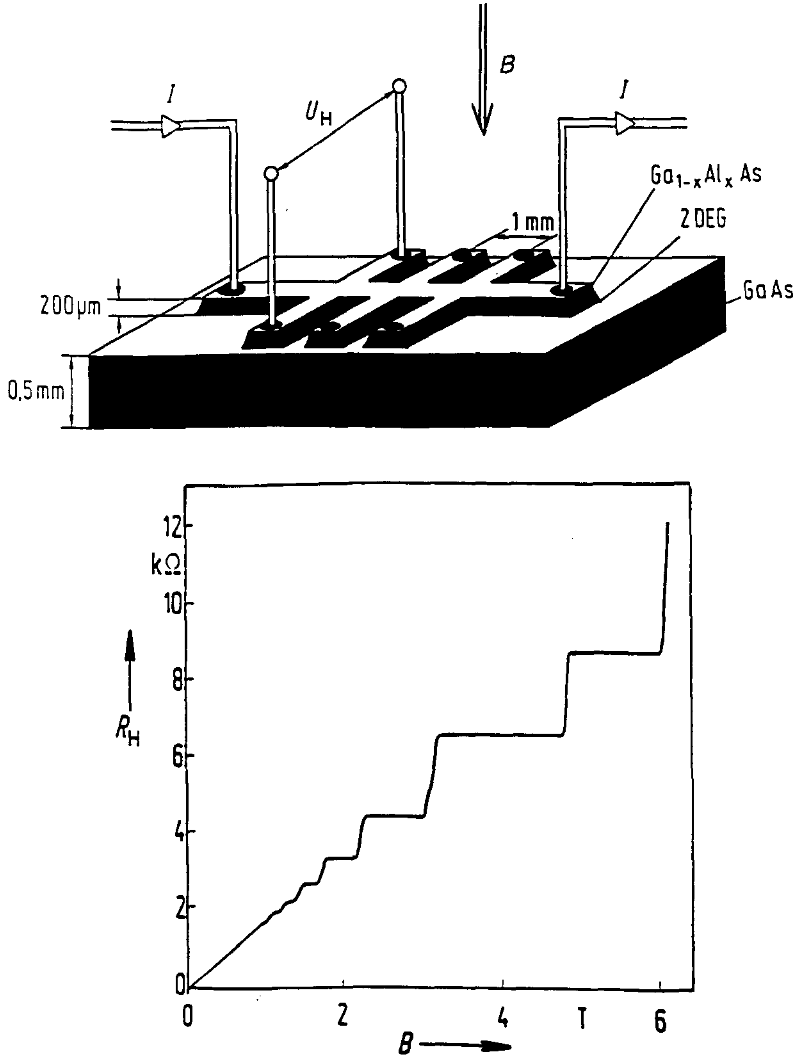
All standard resistances change with time. They also change with temperature and air pressure and show hysteretic behaviour as a result of these changes. Therefore if one looks at the relative positions of the as-maintained units of resistance in various national standards laboratories during the last 30 years, differences between the units in these laboratories can be observed. This is shown in Fig. 11, where the standard maintained at the National Measurement Laboratory (NML), Australia, has been taken as a reference [9].

3.3 Consequences

After the discovery of the two macroscopic quantum effects, it took only a few years until the quantised values could be reproduced with uncertainties much lower than those of the corresponding units. This gave rise to an intense discussion on whether or not there should be changes in the base units of the SI system.

Let us look at our presently defined base units and remember James Clerk Maxwell's challenge in 1870:

> "If we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions or the motion, or the mass of our planet, but in the wavelength, the period of vibration and the absolute mass of these imperishable and unalterable and perfectly similar molecules."



$$\boxed{R_H(i) = \frac{1}{i}\,\frac{h}{e^2}}$$

Fig. 10:
Reproduction of the ohm via the quantum Hall effect:
a) GaAs-$Ga_{1-x}Al_xAs$ heterostructure with electrodes for the measurement of the Hall resistance (2DEG is the two-dimensional electron gas at the interface of the two layers)
b) Hall resistance R_H as a function of an applied magnetic field at a temperature of 0,008 K.

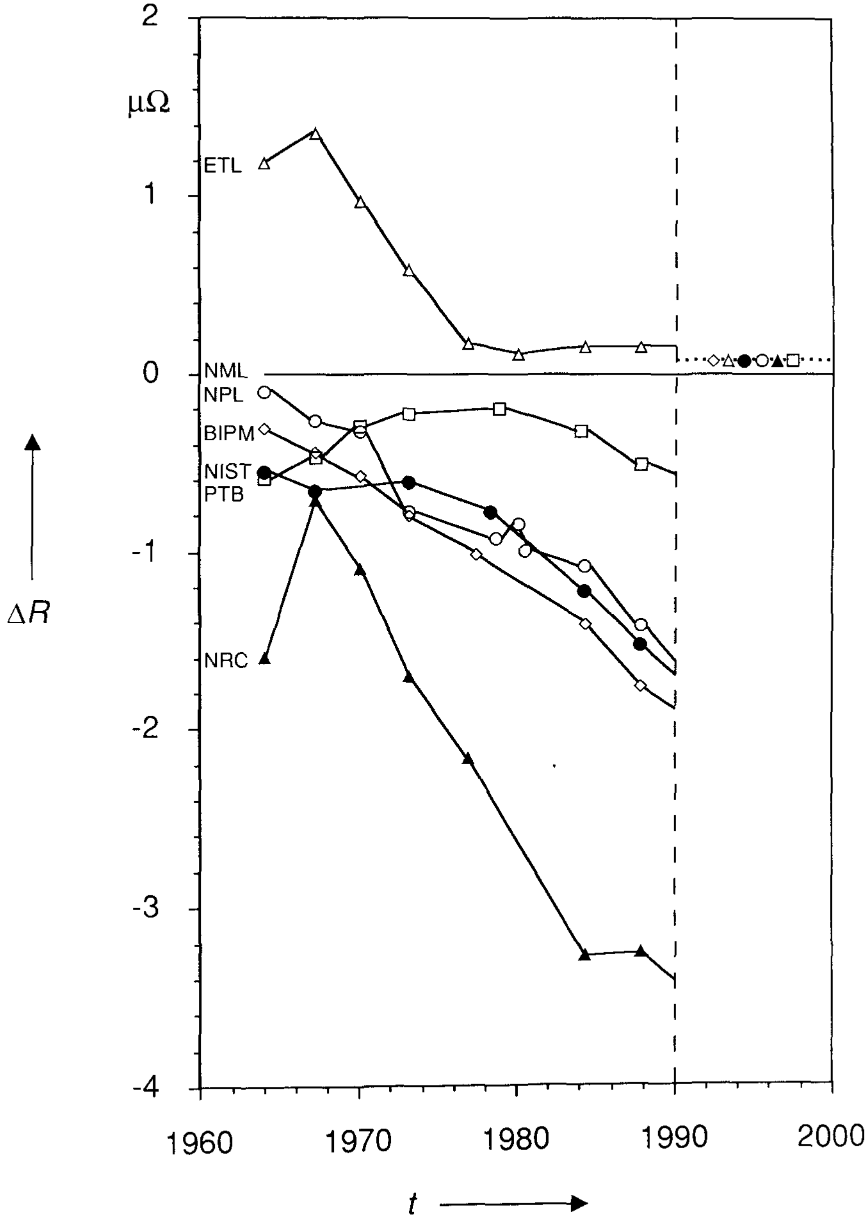


Fig. 11
Changes with time of the as-maintained units of resistance in various national institutes relative to that at the NML.
(Abbreviations see appendix).

Today, this has been fulfilled for length and time, but clearly not for mass and for all units depending on mass like all electric units. The temptation to use the new macroscopic quantum phenomena for a new set of base units was great.

However, the executive bodies of the Metre Convention decided not to change the SI system, but to recommend exact conventional values for the characteristic fundamental constants appearing in the equations for the Josephson voltage and the quantised Hall resistance. The following values should be used for the maintenance of the volt and the ohm:

instead of $2e/h$:	$K_{J\text{-}90} = 483\,597{,}9$ GHz/V	(Josephson constant)
instead of h/e^2:	$R_{K\text{-}90} = 25\,812{,}807\ \Omega$	(von Klitzing constant)

The advantages of this agreement, which has been accepted by all members of the Metre Convention, are:

- the electric units are reproducible to a high degree ($\leq 10^{-9}$),
- international uniformity leads to the same numerical values when calibrated in different institutes,
- constancy of reproduction with time, and
- at present, the best agreement with the SI value due to the proper choice of the conventional values.

Clearly the uncertainty within the SI system still depends on classical experiments.

4. Metrology Systems to Achieve Traceability

Signing the diplomatic treaty of the Metre Convention in 1875 was the first step towards world-wide use of the same system of units. Since this time great effort has been made by scientists to develop the system. The increase in scientific knowledge, especially in recent years, has made a great impact on metrology. On the other hand, the increase in complexity of modern technology and the growing together of national trade areas to form supranational markets are matched by a constant demand for more accuracy, wider range and greater diversity in measurement standards. This has enabled the executive bodies to build up lines of traceability which connect each standard in use with the relevant international measurement standard. These systems are intended to guarantee that each standard in use in production or international trade is traceable to an SI unit within a known uncertainty.

In order to reach this goal, systems have been set up

on a national basis such as

calibration services for industrial metrology and
verification services for legal metrology;

on a supranational basis regional organisations were founded to serve the arising concentration of markets:

EUROMET in Western Europe,
COOMET in Eastern Europe,
NORAMET in North America,
Asia Pacific Metrology Program in South East Asia,
SIM in Ibero-America

The highest level in this hierarchy is represented by the Metre Convention and its laboratory, the

Bureau International des Poids et Mesures (BIPM)
in Sévres near Paris.

Fig. 12 shows the lines of traceability within the present metrology system supplying every user with the appropriate service. Here the term "primary laboratory" is derived from the definition of primary standards as those having the highest metrological qualities, and the term "secondary laboratory" from secondary standards as those being fixed by comparison with primary standards. Together with the BIPM about ten national laboratories maintain the SI.

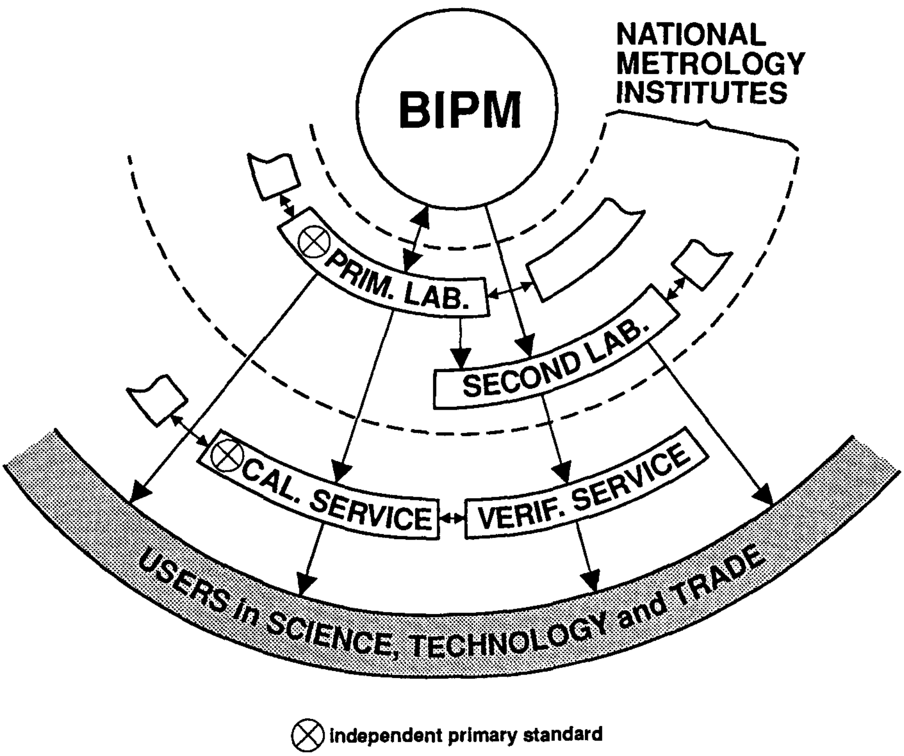


Fig. 12
Lines of traceability within the international metrology system.

However, recent developments have made it possible to establish traceability other than through the traditional hierarchical chain extending from the BIPM to a national metrology institute and ending at the user. To give an example, units like the volt and the ohm which are related to quantum phenomena are reproducible everywhere and at any time; they could be termed as "independent primary standards". Therefore any laboratory which has these standards, in our example based on the Josephson and the quantum Hall effect, has the highest end of the chain of traceability as its own.

5. Conclusion

The principles of metrology have been explained. They start with the development of a system of units on a scientific basis. The SI system was developed for this purpose and internationally accepted and adopted. They end with the well-documented traceability of each measurement standard in use to the defined one.

Electricity has been chosen as an example since it is used everywhere in science, technology and trade. Metrology in electricity guarantees the proper functioning of practically all electrical devices. International agreement on conventional values for characteristic fundamental constants gave rise to independent primary standards based on macroscopic quantum effects, providing high reproducibility of electric measurements worldwide.

At present we can proudly say that the international metrology system meets all demands. It serves science and technology, facilitates national and international trade, and provides good service for every user.

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Appendix:

BIPM Bureau International des Poids et Mesures;
ETL Electronical Laboratory, Japan;
NIST National Institute of Standards and Technology, USA;
NPL National Physics Laboratory, United Kingdom;
NRC National Research Council, Canada;
PTB Physikalisch-Technische Bundesanstalt, Germany.